MATHEMATICS FOR INFORMATICS EDUCATION

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Abstract: *Mathematical education acquires specific knowledge and adopts a mathematical apparatus that has a direct application in solving practical problems. Mathematics contributes to the development of general cognitive and metacognitive skills, abstract, logical, critical, and creative thinking, problem-solving strategies, and decision making, which is useful in all computer science disciplines. Also, problems arising in the entire spectrum of information technology have an increasing influence on mathematics. The paper discusses mathematical areas that need to be studied in all jobs in the field of informatics and the importance of learning mathematics for an information technology student. The authors make a correlation between specific mathematical areas and their application in the appropriate computer science discipline. This paper is designed for those who would like to understand some of the mathematical concepts behind important modern applications in informatics.*

Key words*: mathematics education, informatics*

1. Introduction

Mathematical education is the acquisition of specialized knowledge and the use of a mathematical apparatus that can be used to solve real-world problems. Mathematics aids in the development of general cognitive and metacognitive skills, as well as abstract, logical, critical, and creative thinking, problem-solving tactics, and decision-making, all of which are useful in computer science. Problems occurring throughout the entire range of information technology are also having an increasing impact on mathematics. [1], [3].

In the United States, the Bureau of Labor Statistics (BLS) tracks labor market activities and working conditions.

Employment in occupations related to STEM [science, technology, engineering, and mathematics] is projected to grow to more than 9 million between 2012 and 2022. That’s an increase of about 1 million jobs over 2012 employment levels. [2]

Many of these STEM careers are tied to technology, as shown in table 1. According to the Bureau of Labor Statistics, there will be over 200,000 job openings for applications software engineers between 2012 and 2022.

By 2026, the US Bureau of Labor Statistics predicts that were will be 1.5 million computing jobs but just 400,000 computer science students with the skills to apply for those jobs.

A combination of talents, education, and career-starting education is required of potential STEM workers. Future STEM workers must have good thinking and communication abilities in addition to a strong technical foundation. Success in STEM careers requires the capacity to think about problems in a variety of ways and to convincingly articulate solutions.

STEM employees can use critical and creative thinking to help them find mistakes, obtain essential information, and comprehend how different pieces or systems connect with one another when they're solving problems.

STEM personnel must also possess critical thinking skills in order to generate new and cost-effective solutions. STEM employees that are innovative can take an alternative approach to the problem, for example, by incorporating information from other fields.

Technical writing, public speaking, interpersonal communication, and the ability to easily explain difficult subjects are all examples of communication abilities.

Some goals of mathematics education are listed:

a. Belief that, with understands of its utility, power, and beauty, mathematics penetrates the world around us;
b. Enjoying math and developing patience and perseverance when solving problems;
c. Understanding and knowledge of mathematics' language, symbols, and notation;
d. Developing mathematical curiosity and inductive-deductive reasoning when solving problems;
e. Using mathematics to analyse and solve issues in real-world settings;
f. The development of abstract, logical, and critical thinking and skills;
g. The development of a critical awareness of the use of information and communication technologies.

Table 1. Selected STEM occupations with many job openings, projected 2012–22

Occupation	Job openings, projected 2012–22	Employment		Median annual wage, May 2013	Typical entry-level education[1]
		2012	Projected 2022		
Software developers, applications	218,500	613,000	752,900	$92,660	Bachelor's degree
Computer systems analysts	209,600	520,600	648,400	81,190	Bachelor's degree
Computer user support specialists[2]	196,900	547,700	658,500	46,620	Some college, no degree
Software developers, systems software	134,700	405,000	487,800	101,410	Bachelor's degree
Civil engineers	120,100	272,900	326,600	80,770	Bachelor's degree
Computer programmers	118,100	343,700	372,100	76,140	Bachelor's degree
Sales representatives, wholesale and manufacturing, technical and scientific products[2]	111,800	382,300	419,500	74,520	Bachelor's degree
Network and computer systems administrators	100,500	366,400	409,400	74,000	Bachelor's degree
Mechanical engineers	99,700	258,100	269,700	82,100	Bachelor's degree
Computer and information systems managers[3]	97,100	332,700	383,600	123,950	Bachelor's degree
Industrial engineers	75,400	223,300	233,400	80,300	Bachelor's degree
Architectural and engineering managers[3]	60,600	193,800	206,900	128,170	Bachelor's degree
Web developers	50,700	141,400	169,900	63,160	Associate's degree
Electrical engineers	44,100	166,100	174,000	89,180	Bachelor's degree
Computer network architects[3]	43,500	143,400	164,300	95,380	Bachelor's degree

2. Mathematical Applications in Informatics

What is the importance of mathematics for students studying IT? What are the mathematics topics necessary to study IT? Where are the mathematics and IT skills related?
Computer science employs many different mathematical topics as an interdisciplinary and complex field.
Computer science is the study of computer principles and applications in information processing, hardware and software design, and application usage. Mathematical background is required for a solid comprehension of concepts and information processes.

2.1. Algorithms

In today's environment, an algorithmic way of thinking is an important approach to problem solving.
Algorithmic procedures are used in mathematics at all levels of education (for example, addition, subtraction, multiplication, division, division of natural numbers into prime factors, operations with polynomials, partial integration algorithm, algorithm for applying Newton Leibniz's formula, recursion algorithms, and so on).
Only mathematics-intensive students will certainly completely comprehend the algorithmic method, which is a key method in computer science.
Algorithms are used in almost all areas of information technology. An algorithm is a series of instructions which have to be made in a certain order to obtain the desired outcome (step-by-step procedure).
Algorithms exist independently of programming languages, which means that they can be implemented in a variety of languages.
One of the very first algorithms introduced in a course on algorithms is Euclid’s algorithm to compute the greatest common divisor (GCD) of two numbers.The greatest common divisor (GCD) of two natural numbers, m and n ($m \geq n > 0$) is the largest integer that divides both m and n with no remainder.
The basis of the algorithm is the following fact:

For $m \geq n > 0, GCD(m,n) = \begin{cases} n, if\ n\ divides\ m\ with\ no\ remainder \\ GCD\left(n, remainder\ of\ \frac{m}{n}\right), otherwise \end{cases}$

Here is the GCD function pseudo-code:

```
int gcd(int m, int n) {
  if ((m % n) == 0)
    return n;
  else
    return gcd(n, m % n);
}
```

Recursion is a fundamental concept for problem solving and the creation of efficient algorithms, and also known as recursive thinking. It is an extremely powerful notion in the development of objects or techniques to solve issues i.e. the concept of well-defined self-reference.

Recursion is a programming technique in computer science that uses a function that calls itself until a specific condition is fulfilled, at which time the rest of each repetition is processed from the last one called to the first.

While designing a recursive algorithm for a particular issue, it's helpful to consider the various ways we could subdivide a problem to define problems with the same general structure as the original problem.

Three key rules must be followed by any recursive algorithms:

(1) There must be a base case in a recursive algorithm.

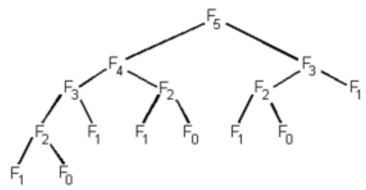
(2) The state of a recursive algorithm must be changed in order for it to return to the base case.

(3) A recursive algorithm must recursively call itself.

Students typically struggle with the concept of recursive thinking/programming since it appears to be circular reasoning.

As an example of a recursive method of computation consider the well known generating the Fibonacci series. The Fibonacci series is defined as follows:

$$fib(n) = \begin{cases} 0, if\ n = 0 \\ 1, if\ n = 1 \\ fib(n-1) + fib(n-2), otherwise \end{cases}$$



A recursive procedure for Fibonacci numbers directly from the definition is given by pseudo-code bellow:

```
Algorithm fib(int n) {
  if (n <= 1)
    return n;                          % base case
  else
    return (fib(n-1) + fib(n-2));      % recursive case
}
```

Remarkably, the first algorithms offered in the course of algorithms and one of the first sequences shown when recursion is introduced are closely connected. The linkage between the Euclidean algorithm and the Fibonacci numbers might be demonstrated in the next level, demonstrating a more advanced link between algorithmic and mathematical thinking. Such examples provide an opportunity for students to become familiar with the terms algorithms, recursion and induction. It allows them to explore the relationship between recursion and iteration in a mathematical sense. In fact, recursive algorithms are the basis for dynamic programming.

From the perspective of data structure, the following are some of the most important kinds of algorithms:

Search Algorithm for looking for a certain item in a data structure; Sort Algorithm to arrange objects in the correct order; Algorithm for inserting an item into a data structure; Algorithm for updating an existing item in a data structure; Algorithm for deleting an existing item from the list.[6]

Algorithms for solving global optimization problems are very important. The goal of global optimization is to find the best possible elements x from a set X according to a set of criteria $F = \{f_1, f_2 \cdots, f_n\}$. Generally, optimization algorithms can be divided in two basic classes: deterministic and probabilistic algorithms (Monte Carlo Algorithms, Evolutionalry Algorithms, Genetic Algorithms...). [15]

In this paper, we will give a quick overview of the most widely used blockchain algorithms.

A blockchain system is fundamentally a distributed system that relies on a consensus algorithm to ensure that dispersed nodes agree on the states of particular data. A consensus algorithm is the essential component that determines how a system acts and how well it performs. [7]

In a nutshell, a blockchain is a series of blocks that hold data. Each block has its own hash as well as the previous block's hash. Before a new block can be added to the blockchain, "Proof of Work" is required. The blockchain database is decentralized and dispersed across several peers.

The blockchain algorithm is as follows:
Step 1: The user initiates a transaction (Transaction data.)
Step 2: The requested transaction is broadcast to a peer-to-peer (P2P) network made up of computers (nodes).
Step 3: Using recognized techniques, the P2P network (nodes) validates the transaction and the user's status.
Step 4: A verified transaction may contain a variety of data (cryptocurrency, records, contracts, or other information).
Step 5. After the transaction has been confirmed, it is joined with the other transactions to form a new block of data for the general ledger.
Step 6: A new block is created by adding it to the blockchain in a permanent and irreversible manner.
Step 7: The transaction is completed.

Figure 1 shown below illustrates what has been explained, as well as how blockchain protects against all forms of corruption. [8]

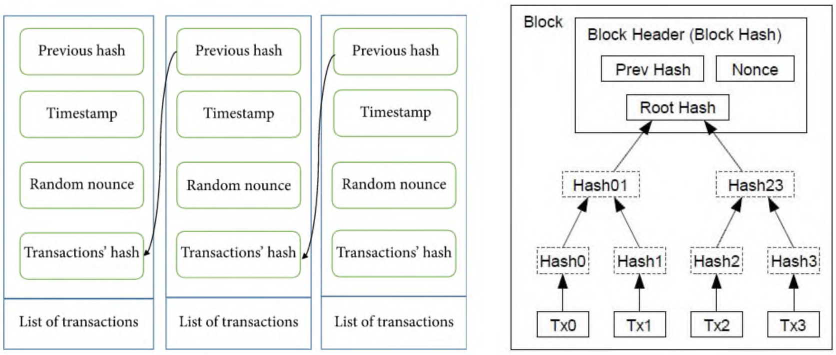


Figure 1. Blockchain illustration.

2.2. Binary Mathematics

One of the most important technological achievements in history is the binary number system. Over the course of 200 years, the system's formalization resulted in the invention of electronic circuits based on logic gates.
Understanding how computer data is represented and programs are encoded in information technology and electronics requires knowledge of binary (base 2) and hexadecimal (base 16) coding.
Students learn to convert between binary, decimal, and hexadecimal formats in undergraduate computer science and computer engineering courses. They will learn how to add and subtract integers in unsigned binary and two's complement representations, as well as how to identify overflow in these operations. When comparing numbers in hexadecimal and decimal forms, students make more mistakes than when comparing numbers in binary or decimal representations. However, most students are able to conduct these operations successfully after finishing these courses.
This is important because Boolean algebra, a subject of mathematical logic, has played a crucial role in the creation of digital electronics. Boolean algebra is used in modern programming languages, despite the fact that it was first introduced in 1847 by George Boole in his book The Mathematical Analysis of Logic.
While expressions in elementary algebra mostly refer to numbers and operators, in Boolean algebra, they refer to logical values such as true and false. Boolean algebra is a branch of mathematics that deals with operations on logical values and includes binary variables of 0 and 1. This course on the college level has provided a framework for engaging binary thinking and understanding binary mathematics.
In all computer machine languages and software, binary mathematics plays a vital role. It is used in every operation that takes place on a computer, from establishing an IP address through network routing.

2.3. College Algebra

Linear algebra, vector algebra, and analytical geometry are all covered in college algebra. Linear algebra is the branch of mathematics that deals with linear equations, determinants, matrices, systems of linear equations, and their matrix representations.
Many natural and other phenomena, such as network flow difficulties, are modelled using linear algebra, which allows for fast computing using such models. In operations research, linear programming is used to solve a variety

of practical problems. To work as a Data Scientist nowadays, you must have a strong understanding of linear algebra.
In computer science, algebra is used to design methods and software for working with mathematical objects, particularly in robotics. Transformation matrices, for example, are used in robotics to shift coordinate systems (see Figure 2) [13].

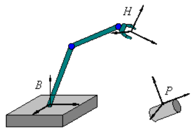


Figure 2. Transformation matrices

A robot's coordinate system is changed to perform a specific activity. There are three associated coordinate systems in the image above: B, the robot's base, specifies the location of the hand, H, the hand, measures distance from the hand, and P, the component the robot must grip, locates points on the cylinder.
The systems are linked in the sense that moving the base B, hand H, or piece P moves the related coordinate systems as well. The primary goal is to establish the position and orientation of the piece P in relation to the robot's hand H so that the hand may grip the piece correctly.
To define the location and orientation of the second coordinate system P with respect to the first coordinate system H, a transformation matrix can be employed. To get the origin and endpoints of the unit vectors (i’, j', k') of the second coordinate system, multiply the transformation matrix T with the first coordinate system (see Figure 3).

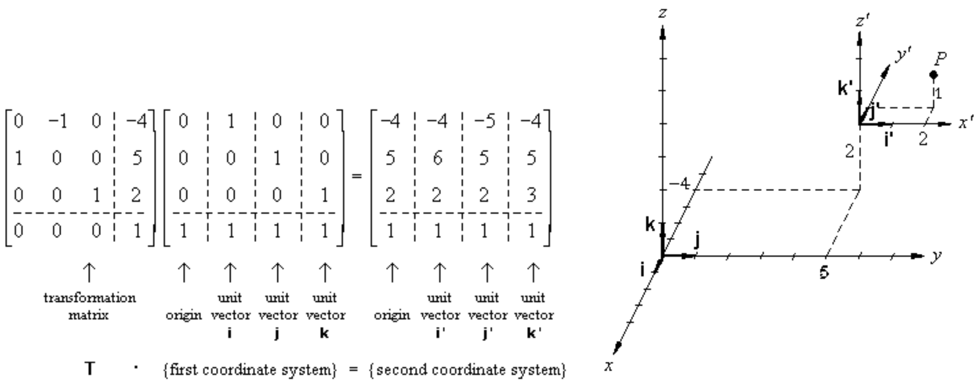


Figure 3. Mathematics behind robotics

These algebraic fields are also employed in computer graphics. The use of vectors and matrices to explain how an item can be rotated, shifted (translated), or made larger or smaller is a beautiful approach to express it (scaled).
Anyone interested in computer graphics should have a strong understanding of algebra. Computer graphics is a hugely popular field, especially in the video game industry.
The x, y, and z coordinates of a 3D graphic object represent its location. The 3D object is translated (moved), scaled (resized), and rotated using the following operations. Addition and multiplication are used to create translation and scale, respectively. The sine and cosine functions are used to rotate objects, necessitating the use of trigonometry.
The object's x, y, and z coordinates can be represented as a 3D vector, and the translation, scaling, and rotation operations can be expressed as a series of matrices (sizes 3x3 or 4x4). Other linear algebra concepts, such as matrix inversion, dot product, and cross product, are also useful.
It can be inferred that a strong foundation in algebra and trigonometry at the high school level is required for a professional approach to computer graphics.
Curve recognition has benefited from algebraic geometry. Many problems are better understood in terms of graph theory, and computational math is vital for building efficient algorithms. [1], [9]

2.4. Statistics

Statistics is a branch of mathematics concerned with the collection, organization, analysis, interpretation, assessment, and presentation of data in computer science. Arithmetic mean, standard deviation, variance, skewness, kurtosis, regression analysis, and analysis of variance are examples of statistical metrics.

Because statistics is used for data processing, data mining, speech recognition, vision and image analysis, data compression, traffic modelling, and even artificial intelligence, statistics is an important aspect of computer science. It's also employed in simulations. To comprehend the algorithms and statistical aspects of computer science, you'll need a background in statistics.
Probability and/or statistics are used in a variety of computer graphics applications (including Virtual Reality and Human-Computer Interaction (HCI)). Furthermore, many computer simulations of the real world include varied probability for the occurrence of a specific action.
Data mining is an interdisciplinary subject of computer science and statistics with the purpose of extracting information from a data collection using intelligent methods and transforming it into an intelligible structure for subsequent use.
Data Science is a multidisciplinary field that integrates statistics, data analysis, and machine learning in order to analyse data and make decisions. [12]

2.5. Calculus

Calculus is the study of continuous changes and the rate at which they occur. Differential calculus and integral calculus are the two elements of calculus. Differential calculus is the study of a quantity's rate of change. The area beneath a curve, or the amount where the change rate is known, is calculated using integral calculus.
Students will be able to apply differentiation techniques to optimization issues and function approximation, as well as implicit and logarithmic differentiation. Students will use appropriate integration methods to determine volume, length, and surface area in this course. Students will also learn about first- and second-order differential equations in this course.
A new instructional mediator is provided between these abstract mathematical notions and physical reality: technology. As a result, the relationship between mathematics and technology is considered natural and spontaneous.
As is well known, this mathematical subject is expressed as a major aspect of scientific knowledge in formal and abstract language, with its grammatical and syntactic principles.
As a result, we believe that symbolism should be introduced in phases such as mental structures, with concept visualization whenever possible, beginning with the initial step of concept development.
We notice that the traditional Calculus learning, which we have characterized as transferable, is based on a symbolic approach. Teachers' oral communication is limited, mostly to the communication of symbols and rules, which are immediately transformed into formulae for executing activities.
Students' cognitive challenges worsen, and they eventually abandon the subject. For example, the teacher's oral and written communication on the concept definition of a function's limit reads as follows:
“It is claimed that the real function f(x) tends to the real number A, for x tends to a, if and only if there exists a sufficiently tiny neighbourhood of a, specified for each of such a neighbourhood it is different from a, and f(x) results as belonging to the neighbourhood of A.”

$$\lim_{x \to a} f(x) = A \Leftrightarrow (\forall \varepsilon > 0)(\exists \delta(\varepsilon) > 0)(|x - a| < \delta \Rightarrow |f(x) - A| < \varepsilon)$$

Understanding of the introduced concept necessitates a thorough decoding of words in a specific mathematical language in order to materialize mental schemes as a result of the entire conceptual process, and so on.
Calculus is utilized in a variety of computer science applications, including graphing and visualization, simulations, problem-solving applications, application coding, statistic solvers, and algorithm design and analysis. PID controllers, for example, are significant components of distributed control systems and predictive control structures; their coefficients are frequently adapted using fuzzy and neural control and set by genetic algorithms.
Knowledge of calculus is an important part of advanced computer graphics. This is one area in mathematics in addition to basic algebra that can open the most doors in computer graphics in terms of mathematical understanding. [14]

3. CONCLUSION

A strong foundation for working with informatics could include a good mathematical education, comprehension of mathematical concepts, and growth of mathematical precision in calculation. The authors feel that mathematics education should be included in the informatics curriculum, as it can help future specialists in the field of software improve their skills more effectively.
Further improvements in vocational mathematics teaching should include mathematical modelling of specific real-world situations (“from the concrete to the abstract” or “from the virtual to the abstract”), the development

of algorithmic [computational] thinking, and the application of logical reasoning to problem-solving situations, and other improvements.

Finally, technological advancements have aided the modern approach to mathematical reasoning, which has resulted in modifications in the curriculum of study programs for future generations of students. ICT, for example, adds additional dimensions to mathematics teaching and learning by assisting students in visualizing particular mathematical concepts. [11] In fact, as math educators, we must change our teaching methods to the new generations called "digital children".

Problem-solving, system design, and an understanding of human behaviour are all part of computer thinking, which is based on basic computer science concepts. Complementing and combining mathematics and technical thinking is computer thinking. Because, like all disciplines, computer science's formal components are based on mathematics, it naturally depends on mathematical thinking.

J. Wing (Wing, 2006) recommended that "we should add computational thinking to every child's analytical capacity in addition to reading, writing, and arithmetic." [10]

The connection between mathematics and IT, as would be explained in the following papers, is essential whenever applicable.

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